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Crackling Dynamics in Material Failure as the Signature of a Self-Organized Dynamic Phase Transition

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We derive here a linear elastic stochastic description for slow crack growth in heterogeneous materials. This approach succeeds in reproducing quantitatively the intermittent crackling dynamics observed recently during the slow propagation of a crack along a weak heterogeneous plane of a transparent Plexiglas block [K. J. Måløy *et al.*, Phys. Rev. Lett. **96**, 045501 (2006)]. In this description, the quasistatic failure of heterogeneous media appears as a self-organized critical phase transition. As such, it exhibits universal and to some extent predictable scaling laws, analogous to that of other systems such as, for example, magnetization noise in ferromagnets.

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Driven by both technological needs and the challenges of unresolved questions in fundamental physics, the effect of material heterogeneities on their failure properties has been extensively studied in the recent past (see [1] for a recent review). So far, many efforts have been focusing on the morphology of fracture surfaces (see [2] for a review). In particular, crack surface roughness was recently shown to exhibit anisotropic morphological scaling features [3,4] that could be understood for brittle materials [4].

Here we will focus our study on the dynamics of cracks. In heterogeneous materials under slow external loading, this propagation displays a jerky dynamics with seemingly random sudden jumps spanning over a broad range of length scales [5]. Such a complex dynamics, also called "crackling noise" [6], has also been suggested from the acoustic emission accompanying the failure of various materials [7] and the seismic behavior accompanying earthquakes [8] characterized by power-law energy distribution with an exponent around 1.3–1.5. These distributions can be qualitatively reproduced in phenomenological models such as fiber bundle models [9] (respectively, random fuse models [10]) which schematize materials as a set of brittle fibers loaded in parallel with random failure thresholds and a rule for load redistribution after each failure event (respectively, as a network of electrical fuses with constant resistance and a randomly distributed threshold). However, these simple approaches yield an exponent around 1.9–2.5 significantly higher than the experimental observations. Moreover, they rely on important simplifications which makes quantitative comparisons with experiments difficult.

We will demonstrate here that this crackling dynamics can be fully reproduced through a stochastic description rigorously derived from linear elastic fracture mechanics (LEFM) [11] extended to disordered materials. In particu-

lar, this model succeeds in reproducing *quantitatively* the intermittent crackling dynamics observed recently during the steady slow crack growth along a weak heterogeneous plane within a transparent Plexiglas block [12]. In this description, quasistatic failure of heterogeneous brittle elastic media can be interpreted as a self-organized critical dynamic phase transition and—as such—exhibits universal behaviors. We will then show how one can use universality and previous calculations performed on different systems belonging to the same universality class, here the motion of domain walls in disordered ferromagnets, to derive predictive laws for the failure of materials.

Theoretical description.—LEFM is based on the fact that—in an elastic medium under tensile loading—the mechanical energy G released as a fracture occurs is entirely dissipated within a small zone at the crack tip. Defining the fracture energy Γ as the energy needed to create two crack surfaces of a unit area, under the quasistatic condition, we assume that the local crack velocity v is proportional to the excess energy locally released:

$$\frac{1}{\mu} v = G - \Gamma, \qquad (1)$$

where μ is the effective mobility of the crack front.

In a homogeneous medium, $\Gamma = \Gamma_0$ is constant, and an initially straight crack front will be translated without being deformed. LEFM allows the determination of the energy released G in any geometry. In particular, for the experimental configuration chosen in Ref. [12], considered here and depicted in Fig. 1, where a crack grows at the interface between two elastic plates by lowering the bottom plate at constant velocity V, G is given by [13]

$$G^0(f(t)) = \frac{1}{3} E \delta^3 \frac{(\Delta_0 + Vt)^2}{[c_0 + f(t)]^4}, \qquad (2)$$

P H Y S I C A L   R E V I E W   L E T T E R S

where $c_0 + f(t)$ is the instantaneous crack length, E the Young modulus, δ the thickness of the lowered plate, Δ_0 the initial opening, and c_0 the initial crack length. Then, considering a slow driving velocity such as $Vt \ll \Delta_0$, one can show from Eqs. (1) and (2) that, after a short transient regime, the crack front propagates at a constant velocity:

$$f(t) \simeq v_m t, \quad \text{with} \quad v_m = \frac{V c_0}{2\Delta_0} \quad \text{and} \quad c_0 = \left(\frac{E\delta^3\Delta_0^2}{3\Gamma_0}\right)^{1/4}. \tag{3}$$

In a heterogeneous material, defects induce fluctuations in the local toughness: $\Gamma(x, y) = \Gamma^0(1 + \eta(x, y))$, where the noise term $\eta(x, y)$ captures these fluctuations and x and y denote the coordinates in the propagation and crack front directions, respectively. These fluctuations induce local distortions of the crack front $f(x, t)$, which in turn generate local perturbations in G [14]. To linear order in f, one can show that [15]
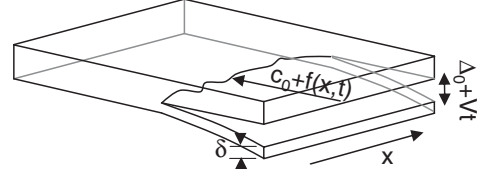

FIG. 1. Crack growth along the inhomogeneous interface between two elastic plates loaded according to the geometry used in Ref. [12]: sketch and notations.

$$G(x, f(x,t)) \approx G^0(\langle f(x,t)\rangle_x)$$
$$+ \frac{1}{2\pi}G^0(\langle f(x,t)\rangle_x)\int_{-\infty}^{\infty} \frac{f(x',t) - f(x,t)}{(x'-x)^2}dx'. \tag{4}$$

Finally, by replacing this expression in Eq. (1), using the expression of G^0 for the homogeneous case, and introducing the quantities defined in Eq. (3), we get

$$\frac{1}{\mu\Gamma^0}\frac{\partial f}{\partial t} = F(t, \{f\}) + \frac{1}{2\pi}\int_{\infty}^{\infty}\frac{f(x',t) - f(x,t)}{(x'-x)^2}dx' - \eta(x, f(x,t)), \quad \text{where} \quad F(t, \{f\}) = \frac{4}{c_0}[v_m t - \langle f(x,t)\rangle_x]. \tag{5}$$

Strictly speaking, this equation describes the interfacial crack growth according to the geometry depicted in Fig. 1. But the very same equation—with different prefactors in the expression of $F(t, \{f\}) = \frac{4}{c_0}[v_m t - \langle f(x,t)\rangle_x]$—would have been obtained for the quasistatic stable crack growth of the in-plane component of the crack front within a three-dimensional solid independently of the tensile loading conditions and the system geometry.

Variants of this equation with *constant* force F have been extensively studied in the past to model crack propagation in solids [14] but also to describe other systems as diverse as interfaces in disordered magnets [16,17] or contact lines of liquid menisci on rough substrates [18,19]. In this case, the interface remains stationary, pinned by the heterogeneities, unless this constant force F is larger than a threshold value F_c. A key feature of these systems is that the so-called depinning transition at F_c belongs to the realm of collective critical phenomena characterized by universal scaling laws [20]. In particular, at F_c, the interface moves through scale-free avalanches, in both space and time.

Equation (5) denotes a rather different situation where the effective force $F(t, \{f\})$ is not constant anymore but given by the difference between the mean front position and the one that would have been observed within the homogeneous case: When $F(t, \{f\}) < F_c$, the front remains pinned, and $F(t, \{f\})$ increases with time. As soon as $F(t, \{f\}) > F_c$, the front propagates, $F(t, \{f\})$ increases, and, as a consequence, $F(t, \{f\})$ is reduced until the front is pinned again. This retroaction process keeps the crack growth close to the depinning transition *at each time*, and, within the limit $v_m \to 0$ and $c_0 \to \infty$, the system remains at the critical point during the whole propagation, as for self-organized critical systems [21].

Spatiotemporal intermittent dynamics.—Predictions of this stochastic description are now confronted by the experimental observations reported in Ref. [12]. Using a fourth-order Runge-Kutta scheme, Eq. (5) is solved for a front $f(x, t)$ propagating in a 1024×1024 uncorrelated 2D random Gaussian map $\eta(x, y)$ with zero average and unit variance [note that the crackling dynamics statistics detailed thereafter do not depend on the precise distribution of η as far as it remains short-range correlated]. The parameter $\mu\Gamma_0$ was set to unity, while the two parameters c_0 and v_m were varied from 2.5 to 250 and 10^{-2} to 5×10^{-1}, respectively. In order to characterize the scaling features of the crack front local dynamics, we adopt the analysis procedure recently proposed in Ref. [12] and compute at each point (x, y) of the recorded region the time $w(x, y)$ spent by the crack front within a small 1×1 pixel^2 region as it passes through this position. A typical gray-scale image of this so-called waiting-time map are presented in Fig. 2(a). The numerous and various regions of gray levels reflect the intermittent dynamics and look very similar to those observed experimentally [Fig. 1(b) of Ref. [12]]. From the inverse value of the waiting-time map, we compute a spatial map of the local normal speed velocity $v(x, y) = 1/w(x, y)$ of the front as it passes through the location (x, y). Avalanches are defined as clusters of velocities v larger than a given threshold $v_c = C\bar{v}$, where $\bar{v}$ denotes the crack velocity averaged over both time and space within the steady regime. Their area is power-law distributed with an exponent $\tau_0 = -1.65 \pm 0.05$ [Fig. 2(c)]. These clusters exhibit morphological scaling features since their width—measured along the direction of crack growth—is shown to go as a power law with respect to their length—measured along the direction of the crack front—with an exponent $H = 0.65 \pm 0.05$
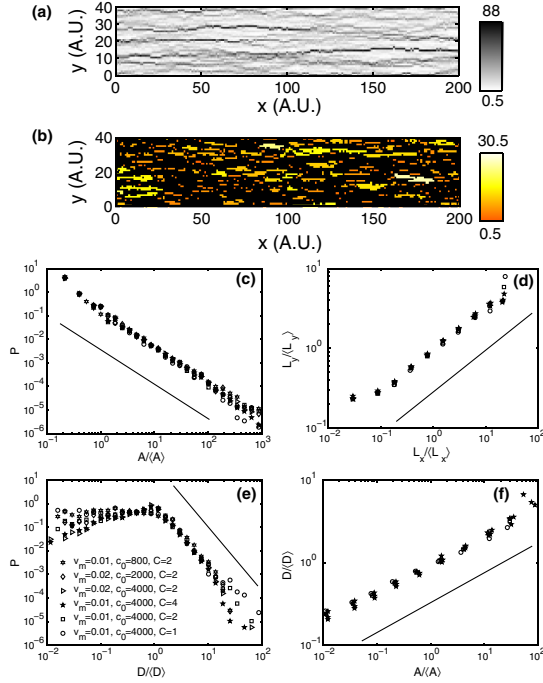
FIG. 2 (color online).   (a) Typical gray-scale map of the
waiting-time matrix $w(x, y)$ obtained from the solution of
Eq. (5) with $c_0 = 4000$ and $v_m = 0.01$. (b) Spatial distribution
of clusters corresponding to velocities $v(x, y) > C\bar{v}$, with $C =$
4. The quantity $\bar{v}$ refers to the crack velocity averaged over both
time and space within the steady regime. The cluster duration is
given by the cluster color according to the color scale given in
the inset. The distribution of the cluster area A, the scaling
between the width L_y and the length L_x of clusters, the
distribution of the cluster duration D, and the scaling between
D and A are plotted in (c), (d), (e), and (f), respectively. The
various symbols correspond to various values of v_m, c_0, and C
as specified in the inset in (e). The straight lines correspond to
$P(A) \propto A^{-\tau_0}$, $L_y \propto L_x^H$, $P(D) \propto D^{-\alpha_>}$, and $D \propto A^\gamma$, with $\tau_0 =$
-1.65, $H = 0.65 \pm 0.05$, $\alpha_> = -2.1$, and $\gamma = 0.4$.

[Fig. 2(d)]. All of these results are in perfect agreement
with those measured experimentally in Ref. [12]. Contrary
to what was conjectured in Ref. [12], H is *significantly
different* from the value of the roughness exponent $\zeta \simeq$
0.39 expected for the interface at the depinning point of
Eq. (5) [14,22]. It is worth mentioning that recent experi-
mental results [23] report a roughness exponent $\zeta \simeq 0.35$ at
a large scale, in agreement with this theoretical prediction.

To complete the characterization of the avalanche sta-
tistics, we measure a new observable, the avalanche dura-
tion—defined as the difference between the times when
the crack front leaves and arrives at the considered ava-
lanche cluster [Fig. 2(b)]. The distribution of the avalanche
duration is plotted in Fig. 2(e). For durations D smaller
than the average avalanche duration $\langle D \rangle$, this distribution
clearly depends on v_m, c_0, and the clipping threshold C,
while for $D \gg \langle D \rangle$ all of the distributions seem to collapse
onto a single power-law behavior characterized by an
exponent $\alpha_> = -2.1 \pm 0.2$ [Fig. 2(e)]. Finally, the mean
avalanche duration is found to go as a power law with the

mean avalanche size, characterized by an exponent $\gamma =$
0.4 ± 0.05 [Fig. 2(f)].

Spatially averaged dynamics.—As for other critical sys-
tems, very different physical systems that belong to the
same universality class will display similar scaling behav-
ior with the same scaling exponents. In particular, it was
shown that the Barkhausen noise [16,17] accompanying
the motion of domain wall driven by an external magnetic
field through soft disordered ferromagnets can be de-
scribed by an equation with a shape similar to Eq. (5). In
this respect, we analyze the global crack front dynamics
by computing the spatially averaged instantaneous velocity
$v(t) = \langle \frac{\partial f}{\partial x}(x, t) \rangle_x$ of the crack interface, as in Refs. [16,17].
We observe once again a jerky dynamics really similar to
Barkhausen noise [16,17]. We then impose a given refer-
ence level $v_c = C_{\max}(v)$ and define bursts as zones where
$v(t)$ is above v_c. The duration T of a given burst is defined
as the interval within two successive intersections of $v(t)$
with v_c, while the size S is defined as the integral of $v(t)$
between the same points. For the Barkhausen noise, this
analysis leads to power-law distributions $P(T) \propto T^{-\alpha}$ and
$P(S) \propto S^{-\tau}$ [16,17] and a power-law relation between the
duration and size of the avalanches $T \propto S^a$, with critical
exponents that can be predicted by using functional renor-
malization group (FRG) calculations [17,18] leading to
$\tau \approx 1.25$, $\alpha \approx 1.43$, and $a \approx 0.58$. We show in
Figs. 3(b)–3(d) that all of these scaling relations as well
as the values of the exponents are found to be in good
agreement with the ones observed here for the average
interfacial crack growth.

Conclusion.—We have derived a description for planar
crack growth in a disordered brittle material which suc-
ceeds in capturing the statistics of the intermittent crack-
ling dynamics recently observed experimentally [12]. In
particular, we have shown that material failure appears as a
critical system, where the crack front progresses through
scale-free avalanches, a signature of a dynamic depinning
phase transition. As for other critical systems, microscopic
and macroscopic details do not matter at large length and
time scales, and this simple linear elastic stochastic de-
scription contains all of the ingredients needed to capture
the scaling statistical properties of more complex failure
situations. This conjecture is strongly supported by a recent
analysis [24] that has shown that the laboratory experiment
described here of crack growth within Plexiglas under
tension shares many characteristics with geological
faults—although this latter results from external shear
loading—since they exhibit seismicity catalogs with simi-
lar statistical features: time of occurrence, epicenter loca-
tion, and energy parameter. FRG calculations performed
on a different system belonging to the same universality
class—the motion of domain walls in disordered ferro-
magnets—have been used to predict scaling laws for the
statistics of the spatially averaged instantaneous crack
velocity. Experimental checks of these predictions are
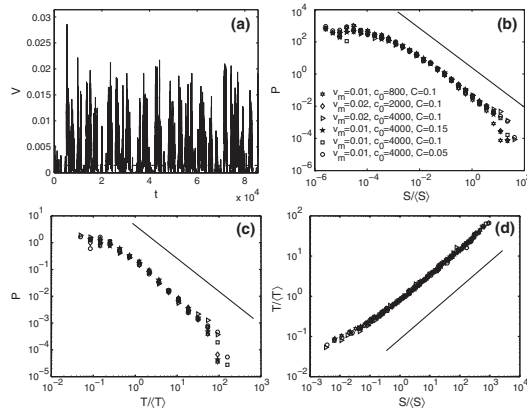currently under progress [25]. Let us note finally that this

FIG. 3.   (a) Typical time evolution of the spatially averaged crack front velocity $v(t) = \langle v(t, x)\rangle_x$. The dashed-dotted horizontal line corresponds to $v_r = C_{\max}(v)$, with $C = 0.1$ used to define the bursts. Distribution of the normalized burst duration T, normalized burst size S, and the scaling of T with S are plotted in (b), (c), and (d), respectively. The various symbols correspond to various values of v_m, c_0, and C as specified in the inset in (b). In these three graphs, the straight lines correspond to $P(T) \propto T^{-\alpha}$, $P(S) \propto S^{-\tau}$, and $T \propto S^a$ with the critical exponents $\alpha = 1.43$, $\tau = 1.25$, and $a = 0.58$ predicted by the FRG approach.

description has been derived within the quasistatic approximation. To understand how to include the dynamic stress transfers through acoustic waves occurring as a *dynamically* growing crack is interacting with the material disorder [26] will represent interesting challenges for future investigations.

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