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Quasi-brittle fracture of heterogeneous materials: a nonlocal damage model

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Abstract

A time-dependent nonlocal continuous damage model is developped in order to describe the transition to failure in quasi-brittle materials. It is based on an energetic approach and takes into account the interactions between microstructural elements through an integral nonlocal formulation in which a microscopic length is included. Using a phase-field approach, the damage evolution law of each point of the system is obtained. The model is applied to a fiber bundle and we study the influence of the internal length on the temporal and spatial organization of damage prior to complete failure. This model captures qualitatively various features of quasi-brittle fracture as observed in experiments and allows for a rigorous analysis of the energy dissipated during the failure process.

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Keywords: Fiber bundle; damage; nonlocal approach; quasi-brittle fracture; intermittency

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1. Introduction

Quasi-brittle fracture of heterogeneous materials occurs by microcracking of the medium. It is characterized by complex temporal and spatial organizations of microcracks that produces both intermittency of damage events and localization (Bažant et al. (1998), Alava et al. (2006)). The present study aims at investigating the origin of these mechanisms by considering a simplified description of the failure process that takes into account microscale material heterogeneity and interactions between elements of the system. To do so, we consider a fiber bundle model with a time-dependent damage law. A nonlocal integral approach (Pijaudier-Cabot et al. (1987)) is used and allows us to control the interactions between fibers of the bundle: damage of a given fiber affects the other fibers over a distance fixed by an internal length. The heterogeneities of the material are also well varied using a statistic distribution of fracture energy of the fibers, which allows for exploring both brittle and quasi-brittle failure behaviors.

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2. Model

2.1. Presentation of the model

We consider a one-dimensional fiber bundle modeled as a continuous system. Fibers are indexed by their position x and distributed equidistantly in space over the range $[0; L]$. They are aligned between to rigid plates: the bottom one is clamped whereas the system is driven by a uniform macroscopic displacement Δ applied to the upper plate, in a direction perpendicular to the axis x. We consider a quasistatic process, so that all fibers reach a mechanical equilibrium before a subsequent increase of the macroscopic displacement. We introduce a damage dependent microscopic stiffness defined by

$$k[\overline{d}(x,\Delta)] = k_0[a\overline{d}^3(x,\Delta) - (a+1)\overline{d}(x,\Delta) + 1] \tag{1}$$

where k_0 is the stiffness of the intact fibers taken constant in the bundle, a is a constant taken equal to -0.3 so that individual fibers break in an unstable process: as long as $a < 0$ the failure is unstable and the actual value of a does not affect significantly the response of the bundle, and $\overline{d}$ is the nonlocal damage parameter. This nonlocal field allows us to introduce the interactions between fibers on an internal length R_0, corresponding to a number of $2R$ fibers interacting, via a weight function α:

$$\overline{d}(x,\Delta) = \int_0^L \alpha(x,\xi) d(\xi,\Delta) d\xi \tag{2}$$

where d is the local microscopic damage parameter, ranging from 0 when the fiber is intact to 1 when it is broken, and α is a parabolic function of the distance between fibers at points x and ξ:

$$\begin{aligned} \alpha(x,\xi) &= \alpha_0(x,\xi) / \int_0^L \alpha_0(x,\zeta) d\zeta \\ \alpha_0(x,\xi) &= \begin{cases} \left\langle 1 - \|x-\xi\|^2/R_0^2 \right\rangle^2 & \text{if} \quad \|x-\xi\| \le R_0 \\ 0 & \text{else} \end{cases} \end{aligned} \tag{3}$$

We express the total energy $E(\Delta)$ of the fiber bundle and the loading system as the sum of the elastic energy E^{el}, the fracture energy E^d and the work of the external force W:

$$E(\Delta) = E^{el}(\Delta) + E^d(\Delta) - W(\Delta) = \int_0^L \frac{1}{2}\Delta^2 k[\overline{d}(x,\Delta)]dx + \int_0^\Delta \int_0^L Y_c(x)\dot{d}(x,\Delta)dxd\Delta - \int_0^\Delta F(\Delta)d\Delta \tag{4}$$

where $\dot{d} = dd/d\Delta$. Considering unstable failure of the fibers ($a < 0$) and in the limit of very slow driving rate, damage develops by burts of broken fibers at constant imposed displacement. Therefore the work of the external force does not contribute to the nonlocal damage energy release rate $\overline{Y}$ which can be expressed as

$$\overline{Y}(x,\Delta) = -\delta E^{el}(\Delta)/\delta d = \int_0^L \alpha(x,\xi) Y(\xi,\Delta) d\xi \tag{5}$$

$\overline{Y}$ writes as the convolution of the weight function α with the local energy release rate

$$Y(x,\Delta) = -\tfrac{1}{2}\Delta^2 k_0 \left[3a\overline{d}^2(x,\Delta) - (a+1)\right] \tag{6}$$

Mechanical equilibrium is reached for $\overline{Y} = \delta E^d/\delta d = Y_c$. The actual dynamics of the fiber bundle between successive equilibrium positions is described using a phase-field approach

$$\dot{d}(x,\Delta) = vM\left\langle \overline{Y}(x,\Delta) - Y_c(x) \right\rangle \tag{7}$$

where the rate of damage $\dot{d}$ is proportional to the driving force $\delta E/\delta d$ as considered for example in brittle failure (Bonamy et al. (2008)), v is the loading speed, M is a material constant and $Y_c(x)$ the fracture energy of the fiber located at x. The term in the brackets is non-zero if strictly positive.

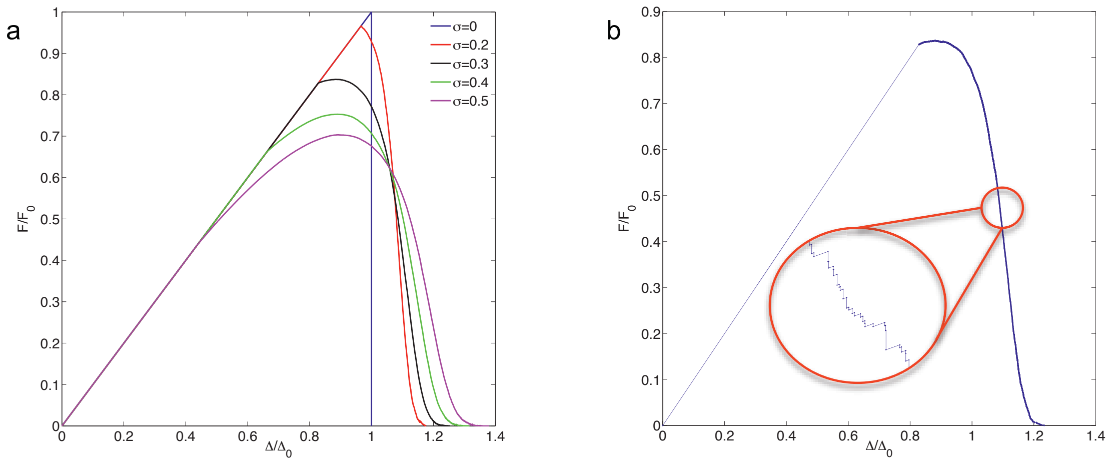

Fig. 1. (a) normalized macroscopic force F/F_0 versus normalized macroscopic displacement Δ/Δ_0 for different values of the level of disorder σ averaged over 50 configurations of the material disorder for $R = 5$; (b) $\sigma = 0.3$, zoom of the mechanical response exhibiting intermittency of damage events.

2.2. *Numerical integration*

Thresholds Y_c values are drawn from a uniform distribution of mean value $< Y_c >=1$ with fixed standard deviation σ that controls the intensity of disorder, i.e. the level of heterogeneity in the system.
To solve the evolution equation (7) we use an explicit discretization scheme that allows for the calculation of the damage field:

$$d(x, \Delta + \delta\Delta) = d(x, \Delta) + vM \left\langle \overline{Y}(x, \Delta) - Y_c(x) \right\rangle \delta\Delta \qquad (8)$$

where $\delta\Delta$ is small enough to assure convergenge of the numerical response of the system. In the following a bundle of $N = 20000$ fibers is considered.

3. Macroscopic failure behavior

We first investigate the macroscopic mechanical response of the system at fixed internal length scale $R = 5$ for different disorder levels σ as shown on Fig.1(a). The macroscopic force and displacement are normalized by the force $F_0 = N\sqrt{2k_0 < Y_c >}$ and displacement $\Delta_0 = \sqrt{2 < Y_c > /k_0}$ of fracture of the system. The curves are obtained by avaraging over 50 realizations of the disorder. As the intensity of disorder σ increases, complete failure is delayed. Varying this parameter allows for the exploration of brittle to quasi-brittle fracture with a more pronounced softening. One observes that the total energy of fracture which coincides with the area under the curves on Fig.1(a) is rather independent of σ, with however negligeable fluctuations resulting from the viscous dissipation during the failure bursts. Despite the apparent smoothness of the mechanical response of the bundle, the system exhibits rather strong fluctuations, as shown on Fig.1(b) where we consider the response of one configuration of the bundle only.

4. Failure bursts analysis

To study the role of interactions in the system, in the following $\sigma = 0.5$ and R is varied from 2 to 2000.

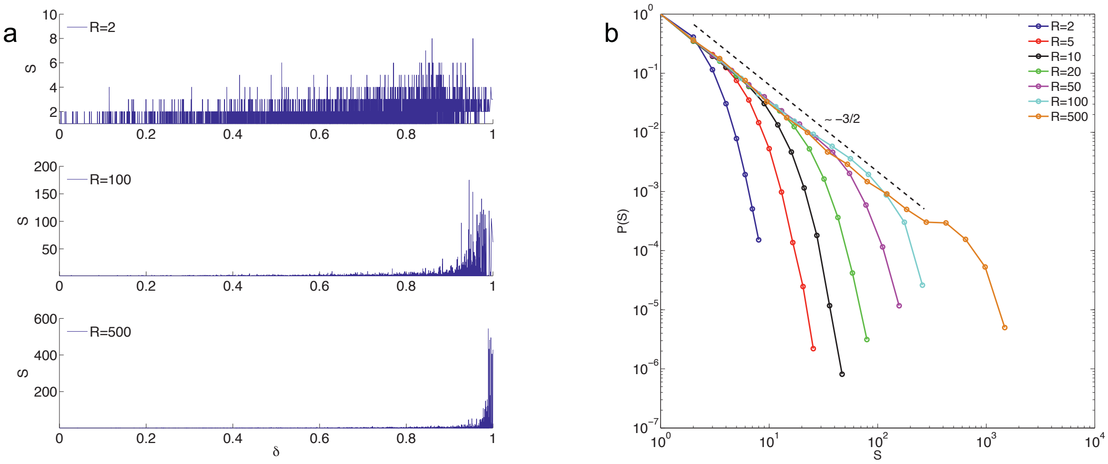

Fig. 2. (a) histogram of avalanche sizes S in terms of number of broken fibers as a function of the distance to failure δ for different internal length scales R; (b) normalized probability density functions of S obtained for different R values and by taking into account all damage events occuring close to failure, for $0.99 \leq \delta \leq 1 = \delta_c$.

4.1. Temporal distribution

As mentioned earlier, the macroscopic response results from intermittency of damage events: macroscopic fracture occurs through the sudden failure of a group of fibers, also referred to as avalanche. To study this phenomenon we define the avalanche size S as the number of broken fibers per displacement increment. The distance to failure is accounted by the parameter

$$\delta = \frac{\Delta - \Delta^{el}}{\Delta^{r} - \Delta^{el}} \tag{9}$$

where Δ^{el} is the displacement at which the weakest fiber fails (end of the elastic regime) and Δ^{r} the displacement at which failure of the whole bundle occurs.
Fig.2(a) shows the avalanche sizes as the bundle approaches full failure: the amplitude of damage events remains of the order of the internal length and the failure process accelerates as macroscopic failure approaches.

We focus on the intermittency of the system close to failure, for $0.99 \leq \delta \leq 1$, and analyze the probability density functions of avalanche sizes for different values of R, as shown on Fig.2(b). For large values of R, it follows a power law of characteristic exponent $3/2$ up to some cutoff size S^* that depends on R: decreasing R shifts S^* toward lower values, eventually suppressing the power law behavior. Note that the largest avalanches are limited to 2-3 times the value of R. As confirmed by the analysis of their spatial extent, damage events are localized within a process zone the size of which is given by the interaction length.

During an avalanche some amount of elastic energy S_{el} stored in the fibers is dissipated in both fracture energy (S_r) and viscous dissipation (S_c). The elastic energy dissipated during an avalanche at displacement Δ is computed from the difference of stored energy before and after the avalanche. The fracture energy of an avalanche is the sum of the fracture energy of the broken fibers. As so, we obtain the expression of the viscous dissipation during an avalanche as $S_c = S_{el} - S_r$. This dissipation that can be shown propotional to $\dot{d}^2$ results from the unstable dynamics of failure events and can be interpreted as the kinetic energy provided to the bundle during the avalanche.
The insets of Fig.3(a) and (b) show the relationship between the rupture energy (resp. viscous dissipation) corresponding to an avalanche of S broken fibers versus the size of the avalanches S, obtained by fixing $R = 500$ and averaging over 50 realizations of the disorder. The rupture energy is linearly proportional to S. However, the viscous dissipation

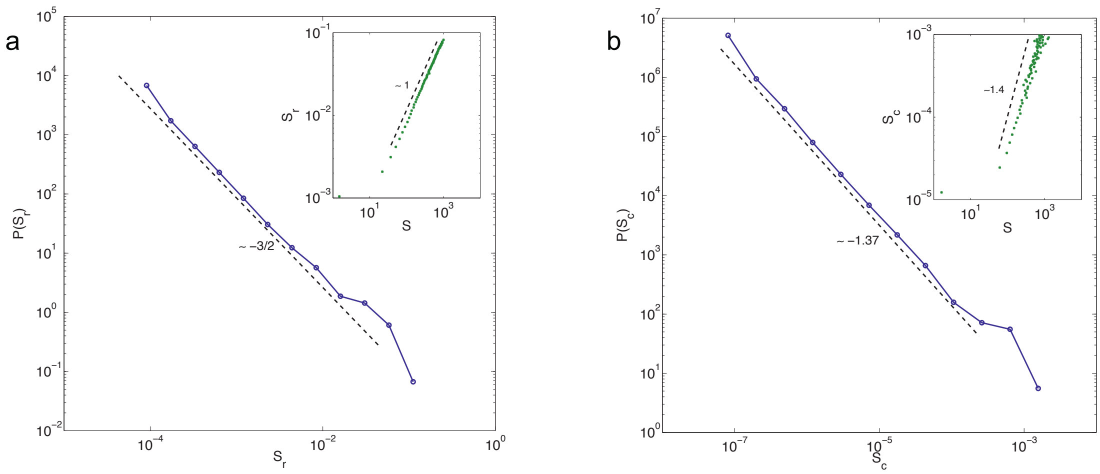

Fig. 3. Statistics of avalanches close to failure for $R = 500$ in terms of dissipated energy: (a) probability density functions of rupture energy of avalanches, the inset shows the linear dependency of the rupture energy per avalanche of size S and the size S of the avalanches; (b) probability density functions of the viscous dissipation during avalanches, the inset shows the power law dependency of exponent 1.4 of the viscous dissipation per avalanche and the size S of the avalanches.

presents a power law dependency with a higher exponent of 1.4. Therefore, under the assumption that the excess of elastic energy release produces the emission of acoustic waves, this finding might indicate that the energy of the acoustic bursts recorded during fracture events are not propotional to their fracture energy. This may have important implications in the interpretation of the acoustic emission largely recorded during fracture experiments.
Focusing on damage events occuring for $0.99 \leq \delta \leq 1$ and using the relation $P(S) \sim S^{\alpha}$ with $\alpha = -3/2$ and $S_r \sim S$ (resp. $S_c \sim S^{1.4}$), we predict $P(S_r) \sim S_r^{-3/2}$ (resp. $P(S_c) \sim S^{-1.35}$), which is in good agreement with the exponents (resp. -3/2 and -1.37) measured directly from simulations that show power law behaviors (see Fig.3).

4.2. Spatial distribution

Spatial evolution of the damage is shown on Fig.4(a) and (b) for different values of R. The temporal evolution is characterized by the density n of broken fibers in the bundle (0 when all fibers are intact and $1 = n_c$ when all fibers are broken). One does not observe any qualitative difference when the interaction length is varied. This observation is confirmed on Fig.5(a) where the variation of the mean number of clusters of broken fibers as a function of the mean size of these clusters is fairly identical in the case of systems with finite interaction length and in the case of an interaction free system where fibers are broken independently, according to their fracture energy (percolation process). Finally, the statistics of clusters of broken fibers is explored through the probability density functions of clusters sizes T as a function of $n_c - n$ (Fig. 5(b)). It displays a cutoff size T^* that grows as the system evolves towards failure. As shown in the inset, this growth is linear with $n_c - n$, following approximately the evolution of the mean cluster size $< T >$, similarly to the behavior of a percolating system.

5. Conclusions

We have proposed a modified fiber bundle model that accounts for the transfer of elastic energy stored in loaded fibers into fracture energy and viscous dissipation. A strong intermittency of the failure process is observed for large internal length scales R only. The avalanche sizes distribution is characterized by a power law with exponent $3/2$. We also characterize this intermittency in terms of bursts of fracture energy and viscous dissipation released during failure events.
The introduction of a finite interaction length does not affect qualitatively the spatial organization of damage events

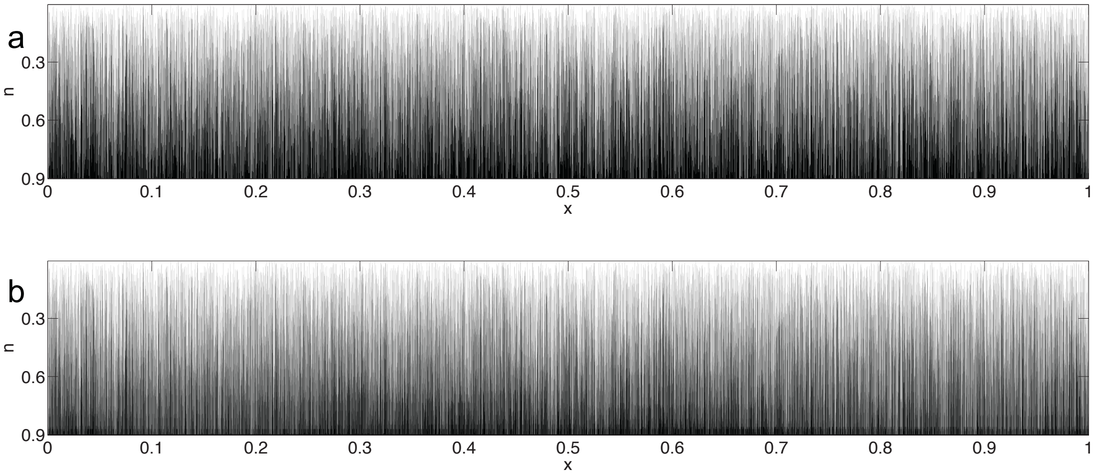

Fig. 4. Spatial evolution of the damage $d(x, n)$ as a function of position x in the bundle and the density of broken fibers n. The broken fibers are in black and the intact ones in white. (a) $R = 2$; (b) $R = 2000$.

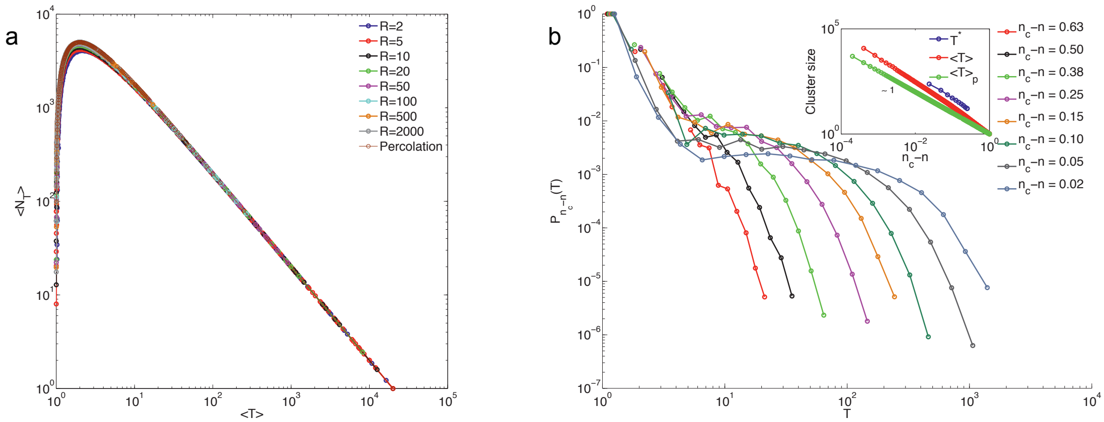

Fig. 5. (a) mean number of clusters $< N_T >$ of broken fibers as a function for their mean size $< T >$ for different values of R exhibiting the very close agreement to percolation theory; (b) normalized probability density functions of broken fibers clusters sizes for different values of the remaining intact fibers density $n_c - n$. The inset shows the evolution of both the mean cluster size $< T >_p$ for a percolating system and for a system with finite internal length scale $R = 5$ ($< T >$) and the cutoff size T^* as a function of $n_c - n$ for the latter one.

which mostly behave as a percolating system for all R values. Application of our approach to two-dimensional systems of fibers may lead to a more complex spatial behavior and in particular to the emergence and growth of a correlation length as the system evolves toward complete failure.

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