Crack growth in brittle heterogeneous materials

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1. Introduction

The effect of materials heterogeneities onto their failure properties remains far from being understood (see [1] for a review of recent progresses in this field). In particular, in heterogeneous materials under slow external loading, cracks growth often displays a jerky dynamics, with sudden jumps spanning over a broad range of length-scales. Such a complex dynamics – also referred to as *crackling noise* [2] was also suggested from the acoustic emission accompanying the failure of various materials [3,4,5] and - at much larger scale - the seismic activity associated to earthquakes [6]. This intermittent dynamics can be qualitatively reproduced in simple numerical models such as Fiber Bundle Models (FBM) (see e.g. [7] for a review) or Random Fuse Models (RFM) (see e.g. [8] for a review) that schematize the material as a set of brittle fibers/network of electrical fuses with randomly distributed breakdown points. However, these descriptions remain phenomenological and fail to reproduce *quantitatively* the intermittent dynamics.

Another approach - pioneered by Gao and Rice [9] and latter extended by Schmittbuhl el al [10] and Ramanathan et al. [11] - consists in extending the standard Linear Elastic Fracture Mechanics (LEFM) of homogeneous materials to the case of heterogeneous media by considering a random field of toughness. In this class of models, the competition between the destabilizing effect of toughness disorder and the smoothing effect of the crack front elasticity makes the onset of crack propagation to appear as a critical depinning transition. This approach succeeded to account for the effective toughness distribution in brittle disordered materials [12] or the large scale morphological scaling features of post-mortem fracture surfaces [13].

We will show here how this approach can be extended to reproduce the crackling dynamics observed during slow stable crack growth in brittle heterogeneous materials. In this model, the slow failure appears as a *self-organized* critical dynamic phase transition and, as such, exhibit universal – and to some extend predictable – statistics and scaling laws. This description succeeds in reproducing quantitatively the intermittent crackling dynamics observed experimentally during the slow propagation of a crack along a weak heterogeneous plane of a transparent Plexiglas block [14].

2. Derivation of a linear elastic *stochastic* model

Let us consider the situation depicted in Figure 1(left) of a crack front that propagates within a 3D elastic solid. Provided that the motion is slow enough, the local velocity of a point $M(z, x = f(z,t), y = h(z, x = f(z,t)))$ is proportional to the excess energy locally released $G(M) - \Gamma(M)$. Material heterogeneities are then modeled by introducing a random component into the fracture energy: $\Gamma(M) = \Gamma^0 (1 + \eta(z,x))$ where η is a short range correlated random term with zero mean and constant variance. This induces distortions of the front, which in turn generates perturbation in G(M). To first order, the variations of G(M) depends on the in-plane distortions f(z,t) only [9,15]:

$$G(M) = G^0 (1 + \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{f(z) - f(z')}{(z - z')^2} dz') \tag{1}$$

where G^0 denotes the reference mechanical energy release which would result from the same loading with a straight front at the same mean position. The equation of motion then reduces to the one of an interfacial front propagating within a 2D random media [9,10,11]:

$$\frac{1}{\mu} \frac{\partial f}{\partial z} = G^0 - \Gamma^0 + \frac{G^0}{2\pi} \int_{-\infty}^{\infty} \frac{f(z) - f(z')}{(z - z')^2} dz') + \Gamma^0 \eta(z, f(z,t)) \tag{2}$$

Where μ is the effective mobility of the crack front. This equation has been extensively studied in the past. It was shown to describe systems as diverse as interfaces in disordered magnets [16,17] or contact lines of liquid menisci on rough substrates [18,19]. In particular, it was shown to exhibit a so called depinning transition controlled by the "force" $F = G^0 - \Gamma^0$. When F is smaller than a given threshold F_c, the front is trapped by the heterogeneities of the

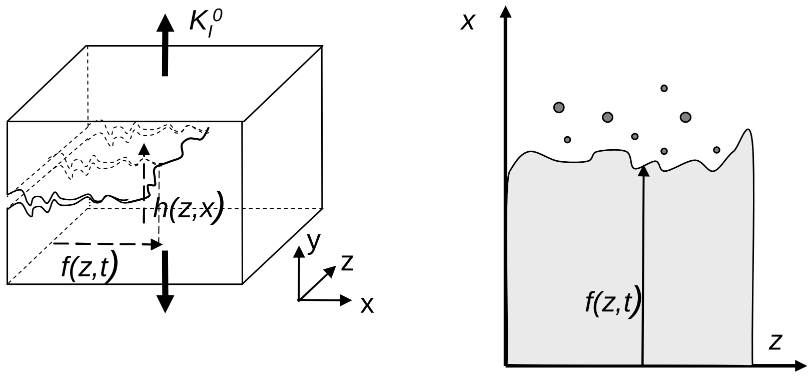

Figure 1. Sketch and notations of a crack front propagating in a 3D heterogeneous material (Left). To first order, the equation of motion involves the in-plane component *f(z,t)* of the crack front and can be reduced to the one of a planar crack front propagating within a 2D material (Right).

material and its velocity is null everywhere. For F larger than F_c, the line is depinned from the last metastable state, and moves with an average velocity $\bar{v} = \langle \partial f / \partial t \rangle$.

Let us now imagine the situation of a stable crack growth under displacement-imposed loading condition. This situation is the one encountered in earthquakes problems where a fault is loaded because of the slow continental drift, or in the experiments described in [14,20] where a crack front is made propagate along the weak heterogeneous interface between two Plexiglas block by lowering the bottom part at constant velocity. Then, G^0 is not constant anymore, but increases slowly with time and decreases with the mean crack length $\langle f \rangle_z$ since the system compliance decreases with $\langle f \rangle_z$ [22]. As a result, provided that the mean crack growth velocity is slow enough and the mean crack length is large enough, one can write $G^0 \approx \text{constant} + ct - k\langle f \rangle_z$ where c and k are constant depending on the precise geometry. Finally, Eq. 2 writes [22]:

$$\frac{1}{\mu\Gamma^0}\frac{\partial f}{\partial z} = ct - k\langle f \rangle_z + \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{f(z)-f(z')}{(z-z')^2}dz') + \eta(z, f(z,t)) \tag{3}$$

The overall force $F(t,\{f\}) = ct - k\langle f \rangle_z$ acting on the front is not constant anymore: When $F(t,\{f\}) < F_c$, the front remains pinned and $F(t,\{f\})$ increases with time. As soon as $F(t,\{f\}) > F_c$, the front propagates, $\langle f \rangle_z$ increases, and, as a consequence, $F(t,\{f\})$ is reduced until the front is pinned again. This retroaction process keeps $F(t,\{f\})$ always close to the depinning transition F_c and the system remains at the critical point during the whole propagation, as for self-organized-critical systems [21].

3. Morphology of the crack front

Eq. 3 is solved using a fourth order Runge-Kutta scheme for a front propagating in a 1024x1024 uncorrelated 2D random gaussian map with zero average and unit variance. The parameter $\mu\Gamma^0$ was set to unity while the two parameters c and k were varied from 10^{-5} to 10^{-3}, and 10^{-3} to 10^{-1}, respectively. Figure 2(left) presents a typical motion of the resulting crack front. The crack propagation is predicted [22] to exhibit an intermittent crackling dynamics and progresses through scale-free avalanches, both in space and time, characterized by universal distributions or scaling laws. In particular, the morphology of the crack front is found to exhibit self-affine scaling features [10] characterized by a roughness exponent $\varsigma \approx 0.39$ [23,24]. In other words, the structure function $G(\Delta z) = \left\langle \left(f(z+\Delta z) - f(z) \right)^2 \right\rangle$ scales as $G(\Delta z) = \Delta z^{2\varsigma}$ with $\varsigma \approx 0.39$ (Fig. 2(right)). This observation is in good agreement with recent observations reported on the large scale roughness of an interfacial crack front propagating along the weak heterogeneous interface between two Plexiglas blocks [20].

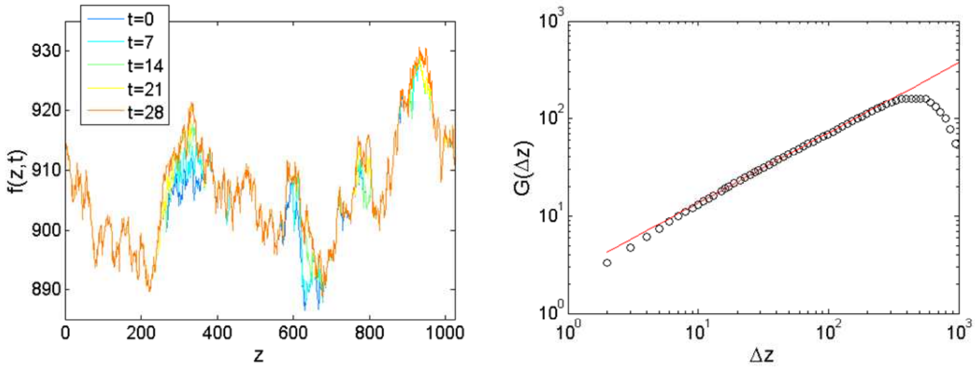

Figure 2. Left: Five successive snapshots of the crack front equally distributed in time in the solution of Eq. (3). Right: Structure function $G(\Delta z)$ as a function of Δz (symbol o) together with a power-law fit (straight line) $G(\Delta z) \sim \Delta z^{2\zeta}$ with $\zeta \sim 0.39$.

4. Crackling dynamics of the mean crack front

As for other critical systems, some features of the distributions and the scaling laws can be predicted. In this context, we analyse the global crack front dynamics by computing the velocity of the crack interface $v(t) = \langle v(z,t) \rangle_z$ spatially averaged along the front direction z (Fig. 3(a)). This signal is extremely jerky, with sudden bursts, signature of an intermittent "crackling noise" dynamics [2]. In order to characterize the statistics of these bursts, we develop the following procedure: (i) we impose a given reference level $v_r = C\bar{v}$ where $\bar{v} = \langle v(z,t) \rangle_{z,t}$; (ii) we define bursts as zones where $v(t)$ is above v_r and (iii) we compute the duration T and the size S of the bursts as the interval between two successive intersections of $v(t)$ with v_r, and the integral of $v(t)$ between the same points, respectively. The distribution of S and the scaling between S and T have been analytically derived for the motion of domains walls in disordered ferromagnets, in the context of Barkhausen effect [16,17]. This analysis leads to a power-law distribution $P(S) \propto S^{-\tau}$ [17], and a power-law relation $T \propto S^{a}$, *with critical exponents that can be predicted using functional renormalisation group calculations* [18] leading to $\tau \approx 1.25$ and $a \approx 0.58$. We show on Fig. 3b and c that these predictions are in good agreement with numerical simulations of Eq. 3 with various values of the parameters *c*, *k* and *C*. This demonstrates how one can make use of the universality associated with this crack growth *self-organized dynamic phase transition* to use previous calculations performed for different systems belonging to the same universality class, here the motion of domain walls in disordered ferromagnets, to derive predictive laws for the failure of materials.

5. Spatiotemporal avalanches dynamics of the crack front.

We then characterized the *local* dynamics of the crack front as predicted from Eq. 4, and compared it to the experimental data performed in university of Oslo on the interfacial crack growth along the weak heterogeneous plane between two Plexiglas blocks [14,20]. We then adopted the analysis procedure proposed in [14] and computed at each point (z,x) of the recorded region the time $w(z,x)$ spent by the crack front within a small $1\times1\,\mathrm{pixel}^2$ region as it passes through this position. A typical gray-scale image of this so-called waiting time map is presented in Fig. 4a. The numerous and various regions of gray levels reflect the intermittent dynamics, and look very similar to those observed experimentally (Fig. 1b of [14]). From the inverse value of waiting-time map, we compute a spatial map of the local normal speed velocity $v(z,x)=1/w(z,x)$. Avalanches are then defined as clusters of velocities v larger than a given threshold $v_c = C\bar{v}$ where $\bar{v}$ denotes the crack velocity averaged over both time and space within the steady regime. For all these avalanches, we computed their size A –defined as their area – and their duration D - defined as the difference between the times when the crack front leaves and arrives to the considered avalanche cluster. A typical map of these avalanches is represented in Fig. 4c. The area is found to be

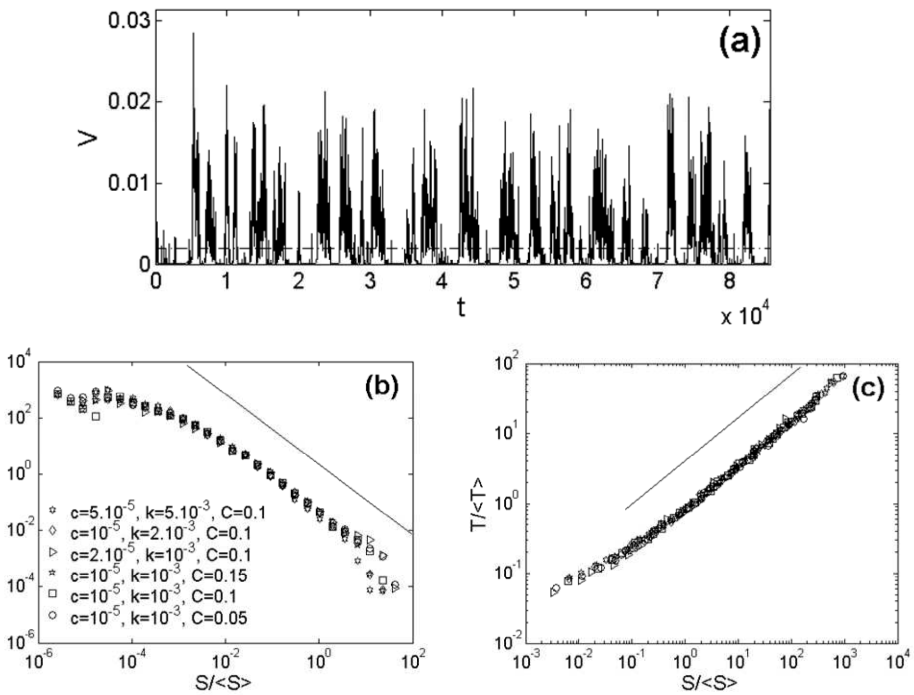

Figure 3. (a): An example of the spatially averaged crack front velocity $v(t)=<v(t,z)>_z$. Bursts are then defined as zone where $v(t)$ is larger than a given threshold $C<v>$ (horizontal dash line) (b) Distribution of the normalized burst size $S/<S>$, and (c) scaling between the burst duration $T/<T>$ and $S/<S>$. The symbols correspond to various values of v_m, c_0 and C. In these two graphs, the straight lines correspond to power-law $P(S)\sim S^{-\tau}$ and $T\sim S^{a}$ with the critical exponents $\tau=1.25$, and $a=0.58$ predicted by RG approach.

power-law distributed with an exponent $\tau_0 = 1.65$ (Fig. 4d), and the mean avalanche duration D is found to go as a power law with A, characterized by an exponent $\gamma = 0.4$ (Fig. 4e). Both results are found to be in very good agreement with experimental observations performed in the group of Oslo.

6. Concluding discussion

We have derived a description for planar crack growth in a disordered brittle material which succeeds to capture the statistics of the intermittent crackling dynamics recently observed experimentally [14,20]. In particular, we have shown that material failure appears as a critical system, where the crack front progresses through scale-free avalanches signature of a dynamic depinning phase transition. As for other critical systems, microscopic and macroscopic details do not matter at large length and time scales and this simple Linear Elastic Stochastic description contains all the ingredients needed to capture the scaling statistical

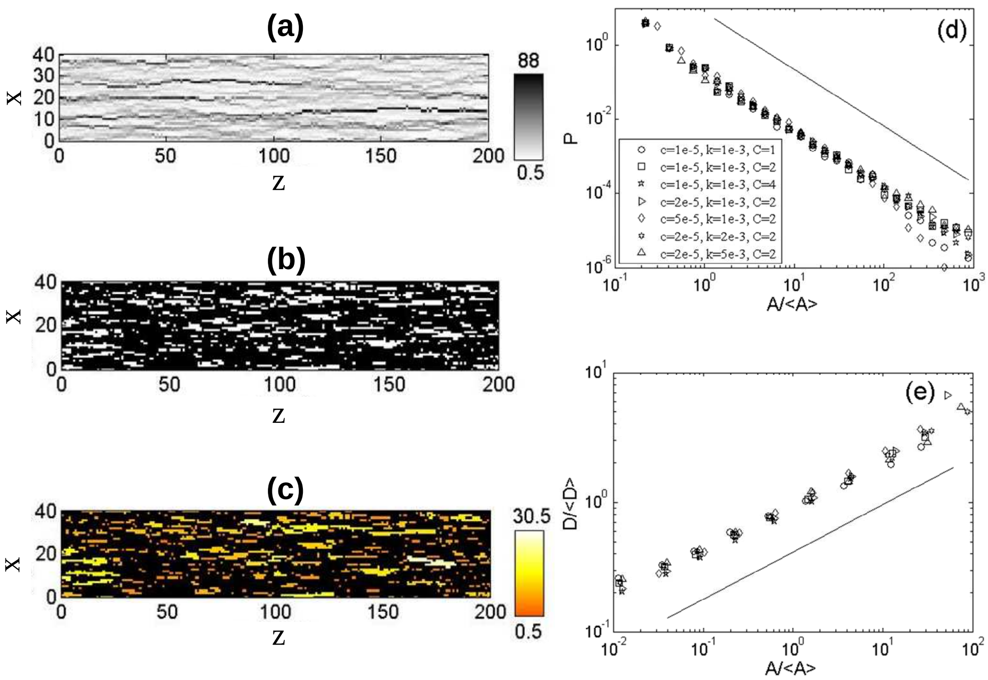

Figure 4. (a) Typical activity map $w(z,x)$ obtained by solving Eq. 4. the gray intensity is proportional to the time spent by the crack front in a given location. (b) Corresponding map of avalanches (in white) defined as clusters where $1/w(z,x)$ is larger than a given threshold. (c) Corresponding map of avalanches coloured according to the colorscale given in inset. (d) Distribution $P(A)$ of avalanche areas together with a power-law fit (straight line) with an exponent $\alpha_0 = -1.65 \pm 0.05$. (e) Scaling between avalanche area and duration together with a power-law fit $D \propto A^{\gamma}$ (straight line) with an exponent $\gamma = 0.4 \pm 0.05$.

properties of more complex failure situations. To compare the predictions of this model with the statistics of earthquakes represents then interesting challenges for future investigations. Work in this direction is under progress.

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