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Crack front fingering during planar crack propagation in highly heterogeneous toughness field

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Abstract

Crack pinning by tougher heterogeneities is in principle an interesting way to toughen brittle materials. To study the impact of highly heterogeneous toughness field, we investigate numerically the propagation of a tensile penny-shape planar crack within an axisymmetric heterogeneous toughness field. In particular, we take into account the large crack front deformations induced by high toughness contrasts. To compute the variations of stress intensity factor along the crack front arising from its progressive deformation, a perturbation approach based on Bueckner-Rice weight function theory is used iteratively. For low enough toughness contrasts, the crack front deforms until reaching an equilibrium shape for which the local stress intensity factor equals the local toughness value at each point of the front. For larger contrasts, however, this equilibrium shape is never reached. Instead, some points of the crack front remained pinned by strong impurities, while some other part of the front advances continuously. The mechanism at the origin of this fingering instability is finally discussed.

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1. Introduction

What is the impact of local material heterogeneities on the propagation of a fracture? It can easily be guessed that the crack advance is slowed down at places where the fracture toughness K_c is higher, so that the geometry of the crack changes during propagation. Since the local loading, that is the stress intensity factor K , depends itself on the geometry through long range elastic interactions, the prediction of the crack shape during propagation is not an easy task. It depends on the level of the toughness contrast but also on the way the toughness varies from place to place, in other words on the spatial toughness distribution.

For small toughness contrasts, linear, first order, perturbation approaches (Rice, 1985) are valid so that it becomes possible to derive some analytical results (see Bonamy (2009); Lazarus (2011) for a review). In particular, it has been shown that two situations may occur depending on the spatial toughness distribution (Roux et al., 2003).

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Nomenclature

\mathcal{F}	A planar closed shape crack.
M	A point along the crack front.
$(a(M), \theta(M))$	Polar co-ordinate of point M .
a	Mean radius of the crack.
a_0	Radius of the initial crack front.
σ	Tensile stress applied at infinity.
$K(M)$	Stress intensity factor along crack front.
$K_c(M)$	Material toughness map along crack plane.
\overline{K}_c	Mean material toughness along crack plane.
$\langle K \rangle$	Effective toughness.
Δ	Relative toughness contrast.
k	Positive wavenumber of toughness fluctuations.

Basically, if the variations of toughness in the propagation direction are moderated, the crack front deforms so that the stress intensity factor reaches the local toughness at each position along the crack front ($K = K_c$ everywhere). It is in particular the case when the toughness map is invariant in the direction of propagation. In contrast, if the spatial fluctuations along the propagation direction are strong, some parts of the front remain pinned on higher toughness positions ($K < K_c$ at some places). Using Gao and Rice (1989)'s terminology, the first situation is coined *regular* and the second *irregular* process in the sequel. In the first case, the heterogeneous material can safely be replaced by an equivalent homogeneous one with effective toughness $\langle K \rangle$ (Patinet et al., 2013), but in the second case, the equivalent toughness can no more be obtained by a simple average and the effective toughness is larger than the spatial average along crack front (Patinet et al., 2013b; Demery et al., 2014).

Our aim here is to study the influence of large toughness contrasts in some, as simple as possible, situations. We restrict to in-plane propagation of a flat crack. In order to focus on the crack front deformations without introducing any boundary effects, we use the finite perturbation method proposed by Bower and Ortiz (1990); Lazarus (2003). Moreover we study a circular crack front (fig. 1) rather than a straight one to avoid any inevitable truncation effects of the discretization. And finally, in this first study, we focus on the toughness contrast effects and avoid to introduce any spatial effects by studying the academic case of an axisymmetric toughness map:

$$K_c(M) = \overline{K}_c [1 + \Delta \cos(k\theta)] \quad (1)$$

In this way, it depends on the position only through the polar angle θ of M so that the toughness is invariant in the directional of propagation. The quantity \overline{K}_c corresponds to the mean value of the toughness, Δ the relative toughness contrast and k the wavenumber. We retrieve that in the limit case of small contrast Δ , regular propagation process is reached but we observe that for large enough toughness contrast, despite the spatial invariance of the toughness in the propagation direction, an equilibrium shape is never reached and a fingering instability occurs. The study of this fingering process is the central aim of this paper.

2. Problem definition

We consider a planar penny shape crack \mathcal{F} of initial radius $a(M)$, embedded in an infinite isotropic elastic medium with heterogeneous fracture toughness properties and loaded in pure mode I through some uniform remote stress applied at infinity σ (see Fig. 1). We assume a quasistatic propagation of the crack front, so that the crack advance at a point M of the front is governed by Irwin's criterion:

$$\begin{cases} K(M) < K_c(M) : \text{no crack advance} \\ K(M) = K_c(M) : \text{possible crack advance,} \end{cases} \quad (2)$$

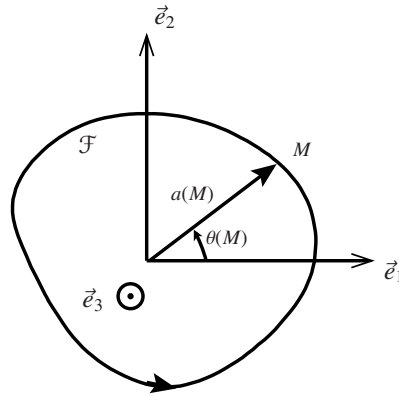


Figure 1. A tensile planar crack in an infinite body under uniform stress σ

where $K(M)$ is the SIF and $K_c(M)$ the toughness at point M given by equation (1).

We suppose that the remote loading σ adapts in order to stay in the quasistatic regime and to ensure crack propagation at least, on some part of the front. This implies that at each moment:

$$\max_{M \in \mathcal{F}} \frac{K(M)}{K_c(M)} = 1 \tag{3}$$

Under the assumption of quasistatic propagation, the problem is to find, for a given toughness map $K_c(M)$, the successive positions of the crack front and the corresponding loading σ . In the sequel, we adapt the method of Lazarus (2003) to our problem with some extension on it. For dimensional reasons, we introduce the dimensionless SIF \widehat{K}

$$K(M) = \sigma \sqrt{a_0} \widehat{K}(M), \tag{4}$$

which depends on crack front shape and crack size and is independent of applied loading. We start from the initial circular crack of radius a_0 for which $\widehat{K}(M) = \widehat{K}_0$ where $\widehat{K}_0(M) = \frac{2}{\sqrt{\pi}}$. The problem is then solved iteratively by successive small perturbations δa of the crack front:

1. Irwin-Griffith propagation law (2) is regularized by some Paris' law with a large Paris exponent (Lazarus, 2003):

$$\delta a(M) = \delta a_{\max} \left(\frac{\widehat{K}(M)}{\widehat{K}_{\max}} \right)^\beta, \quad \beta \gg 1 \tag{5}$$

Advantage of using Paris law is that the crack advance at all steps and discretisation point is directly provided in some explicit way.

2. the dimensionless SIF \widehat{K} is updated by using Rice (1989) first order perturbation formula:

$$\delta \widehat{K}(M_0) = \frac{1}{2\pi} PV \int_{\mathcal{F}} \frac{W(M, M_0)}{D^2(M, M_0)} \widehat{K}(M) [\delta a(M) - \delta_* a(M)] dM \tag{6}$$

In this equation, $W(M, M_0)$ is a dimensionless function of the crack shape, that is updated using the same type of formula, $D(M, M_0)$ is the distance between two points, $\delta_* a(M)$ corresponds to a translation of the crack front and is introduced to ensure the existence of the Principal Value (PV) integral.

3. the critical loading is then obtained by introducing (4) in (3):

$$\frac{\sigma \sqrt{a_0}}{\bar{K}_c} = \max \left(\frac{\widehat{K}_{\max}}{K_c} \right)^{-1} \tag{7}$$

The efficiency of this iterative method, first developed by Bower and Ortiz (1990) arises from the need for the sole 1D meshing of the crack front.

3. Results and discussion

For exploring all possible results, we have run numerical simulations for several toughness maps (1) with different wave numbers k and toughness contrasts Δ . Results from these simulations are showing two kinds of crack propagation behaviour depending on wavenumber and toughness contrast:

- for small toughness contrast $\Delta < \Delta_c(k)$ (fig. 2a):
 At initial stage, only zones of the front where the toughness is weaker start advancing whereas the stronger parts remain pinned. But after this transient phase, the whole front reached Irwin criteria and the front remains in a stationary shape. This situation corresponds to a regular process in the nomenclature introduced in the introduction.
- for large contrast $\Delta > \Delta_c(k)$ (fig. 2b):
 The parts of the front where the toughness is higher never reach the threshold and remain pinned. The crack front never attains a stationary shape. Instead it develops a flower like shape with infinitely growing petals or fingers. It corresponds to an irregular propagation process.

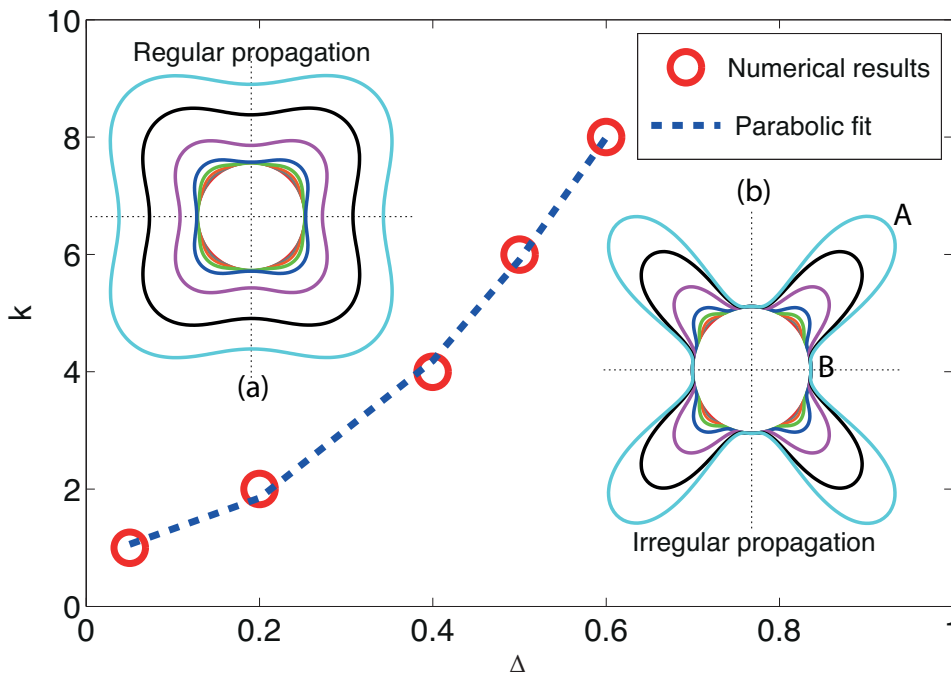


Figure 2. Phase diagram: Regular and irregular propagation. a) example of $k = 4, \Delta = 0.2$, b) example of $k = 4, \Delta = 0.5$.

Fig. 2 represents these two propagation behaviours on a $k-\Delta$ phase diagram. Regions for both behaviours can be separated by a well fitted parabolic relation between wavenumber and toughness contrast which is $k \sim 18.3 \Delta^2 + 0.7 \Delta + 0.97$.

To understand the basic mechanism of such behaviours, we introduce the mean size a of the crack and another dimensionless SIF \tilde{K} by:

$$\widehat{K}(M) = \sqrt{a/a_0} \tilde{K}(M) \tag{8}$$

Whereas \widehat{K} depends on the size and shape of the crack, \tilde{K} depends only on its shape. Fig. 3(a,b) shows the evolution of the normalised \tilde{K}/\tilde{K}_0 during propagation, where $\tilde{K}_0 = \widehat{K}_0 = \frac{2}{\sqrt{\pi}}$ corresponds to a circular crack. Fig. 3(a) and Fig. 3(b) show the evolution in the case of the regular and irregular propagations corresponding to Fig. 2(a,b) respectively. Different colored curves correspond to different stages of the propagation in correspondance with the colors used in Fig. 2(a,b).

Let us denote A a point along the crack front where the toughness is minimum and B where it is maximum (see fig. 2). The advance is thus larger at point A than B . In both cases, at initial stage, the SIF increases at point B and decreases at point A in agreement with the results of Gao and Rice (1987b) valid for small perturbations of a circular crack. In the case of regular crack propagation, the SIF at point B increases until reaching the maximum toughness. At this point the criteria $K = K_c$ is, and further remains, verified all along the crack front. In contrary when the toughness contrast is larger, the SIF at B reaches a peak and then starts decreasing before it had time to reach the maximum toughness. This unloading is linked to the screening induced by the fingers and occurs when the finger size reaches approximately the size of the initial circular crack.

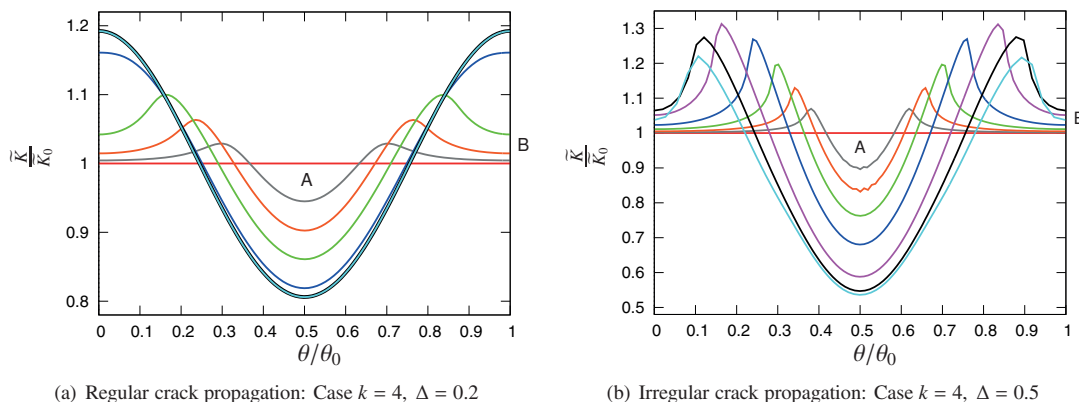


Figure 3. Evolution of \tilde{K}

4. Conclusion

In this paper, we have explored numerically the influence of large toughness contrasts on the crack shape in the absence of spatial changes in the direction of propagation. Whereas for small contrasts, a stationary flower shape is always reached for which $K = K_c$ all along the front (regular situation), for larger contrasts, some zones of the front remains pinned on the higher toughness regions so that the petals or fingers grow infinitely (irregular). By performing systematic simulations for several configurations of the toughness map, corresponding to various combinations of wavenumber k and toughness contrast Δ , we obtained the boundary $\Delta_c(k)$ between the two behaviors: *regular crack propagation* regime for small contrast $\Delta < \Delta_c(k)$ and *irregular crack propagation* regime for large contrast $\Delta > \Delta_c(k)$.

In order to understand the basic mechanism, we have introduced a fully dimensionless SIF parameter \tilde{K} which depends only on the crack front shape. For regular crack propagation, after a transient zone, \tilde{K} reaches Irwin’s threshold at each point of the front and thus the crack front shape reaches a stationary regime. But for larger contrasts,

some points get unloaded due to the screening by the fingers, which ultimately results in constantly growing fingers while some other parts of the front remain pinned.

In summary, for large toughness contrasts, even in the case of weak spatial fluctuations as here, large deformations of the crack front of the same order than the initial crack size may be responsible for complex crack propagation patterns. Further work consists in studying the implication of such irregular crack propagation on the effective fracture properties as defined in Vasoya et al. (2013).

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